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THE SURFACE REJUVENATION MODEL OF WALL TURBULENCE: INNER LAWS FOR u^+ AND T^+

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Abstract—An efficient formulation of the surface rejuvenation model of the turbulent burst process is coupled with the classical approach for the turbulent core to produce inner laws for the dimensionless velocity u^+ and temperature T^+ distributions for fully turbulent flow. The predictions for u^+ are found to be in good agreement with the well known inner law by van Driest. In addition, the predictions for T^+ are shown to be in excellent agreement with experimental data for a wide range of Prandtl numbers.

NOMENCLATURE

g, instantaneous velocity distribution at instant of inrush;
H, instantaneous approach distance;
$\bar{H}$, mean approach distance;
H^+, dimensionless mean approach distance $[H^+ = \bar{H}U^*/\nu]$;
k, thermal conductivity;
Pr, Prandtl number;
P_H, distribution in approach distances;
P_g, distribution in initial profile;
$\overline{q_0''}$, mean wall heat flux;
s, frequency of burst process;
$\bar{s}$, mean frequency of burst process;
$\bar{T}$, mean temperature distribution;
T_i, mean temperature at instant of inrush;
T_0, mean wall temperature;
T^+, dimensionless mean temperature distribution $[T^+ = (T_0 - T\rho c_p U^*/\overline{q_0''}]$;
u, instantaneous velocity distribution;
u^+, dimensionless mean velocity distribution $[u^+ = \bar{u}/U^*]$;
U, mathematical step function;
U_i, mean velocity at instant of inrush;
U_b, bulk stream velocity;
U^*, friction velocity $[U^* = \sqrt{\tau_0 \rho}]$;
y, distance from wall;
y_M^+, dimensionless thickness of hydrodynamic wall region;
y_T^+, dimensionless thickness of thermal wall region.

Greek symbols

ε_H, eddy thermal diffusivity;
ε_m, eddy momentum diffusivity;
γ, $\equiv \bar{H}\sqrt{\bar{s}/\nu}$;
ϕ, age distribution;
θ, instantaneous age of individual burst event;
μ, viscosity;
$\bar{\tau}$, apparent total mean shear stress;
$\bar{\tau}_0$, mean wall shear stress;
$\bar{\tau}_t$, Reynolds stress;
α, thermal diffusivity;
β, $\equiv \bar{H}\sqrt{\bar{s}/\alpha}$.

Superscript

—, mean.

INTRODUCTION

INNER laws have recently been developed for u^+ and T^+ for fully turbulent flow which are based on the nontraditional surface renewal approach in which the burst process associated with wall turbulence is modeled. In accordance with the actual burst process reported in experimental studies [1, 2], the principle of surface renewal stipulates that fluid exchange intermittently occurs between the region immediately adjacent to the wall and the turbulent core. During the period of time between inrush and ejection, unsteady molecular momentum, mass and energy transport is assumed to govern.

The surface renewal modeling concept has been coupled with an eddy diffusivity representation of the turbulent core to obtain convenient inner laws for fully turbulent flow of the form [3]

$$u^+ = \frac{U_i}{U^*}\left[1 - \exp\left(-\frac{y^+}{U_i/U^*}\right)\right] \qquad y^+ < y_M^+ \quad (1a)$$

$$u^+ = \frac{1}{\kappa}\ln y^+ + C \qquad y^+ \geqslant y_M^+ \quad (1b)$$

and

$$T^+ = \frac{U_i}{U^*}\sqrt{Pr}\left[1 - \exp\left(-\frac{y^+\sqrt{Pr}}{U_i/U^*}\right)\right] \qquad y^+ < y_T^+ \quad (2a)$$

$$T^+ = \frac{1}{\kappa}\ln y^+ + A \qquad y^+ \geqslant y_T^+ \quad (2b)$$

where $U_i/U^* \simeq 16$ and $y_M^+ \simeq 46.8$ for $\kappa = 0.4$ and $C = 5.5$; y_T^+ and A are established by equation (2a) by merely requiring continuity in T^+ and dT^+/dy^+. This simple inner law for u^+ lies somewhat below the familiar van Driest [4] and Spalding [5] equations, with a maximum difference being of the order of 6%. Similarly, equation (2) has been found to be in basic agreement with experimental data for Prandtl number Pr up to 5. However, because of the simplifying assumption that the fluid inrush process reaches all the way to the wall, the predictions for T^+ slightly underpredict the data for moderate Pr, with the error increasing with Pr. To eliminate this inadequacy and to extend the applicability of the analysis to high Pr, the fact that the inrush process brings fluid to within varying small distances of the wall must be accounted for.

The earliest model of wall turbulence which includes the effect of the unreplenished layer of fluid which resides at the surface was developed by Harriott [6] in 1962. To develop this surface rejuvenation model, Harriott generated a sequence of random approach distances H and residence times s^{-1} by means of a monumental Monte Carlo technique. This approach was utilized in 1971 to develop predictions for the temperature distribution for high Prandtl number fluids [7]. A more efficient stochastic formulation of this generalized surface renewal model was developed by Bullin and Dukler [8] in 1972 and was applied to turbulent heat transfer by Rajagopal and Thomas [9] in 1974. However, the usefulness of this formulation is severely restricted by the need for extensive numerical iteration. Finally, in 1975 a new approach [10] was set forth for the formulation-solution of the surface rejuvenation model which leads to exact analytical solutions for the mean velocity and temperature distributions within the wall region for fully turbulent conditions. The objective of the present paper is to develop inner laws for u^+ and T^+ by coupling this efficient formulation of the surface rejuvenation model of the turbulent burst phenomenon with the classical eddy diffusivity formulation for the turbulent core.

MATHEMATICAL FORMULATION—SOLUTION

The turbulent burst process is pictured in Fig. 1 for a single cycle. Such events are envisioned to occur over the entire surface with the burst frequency s and the approach distance H associated with the inrush phase varying randomly about mean value $\bar{s}$ and $\bar{H}$. To transform this physical picture of wall turbulence into a manageable mathematical format, certain modeling assumptions are employed. The mathematical formulation involves (1) an analysis of the instantaneous transport within the wall region during the period between inrush and ejection, (2) an evaluation of the spatial mean transport properties associated with the overall process, and (3) specification of key modeling parameters. The surface rejuvenation formulation–solution for momentum transfer is developed in this section. This formulation follows the pattern of the analysis for energy transfer which was developed in reference [10].

Instantaneous transport

Assuming that the convective and pressure gradient effects are small within the wall region for fully turbulent conditions, the unsteady momentum transfer within the wall region for the period between inrush and ejection takes the form

$$\frac{\partial u}{\partial \theta} = \nu \frac{\partial^2 u}{\partial y^2} \tag{3}$$

$$u = U_i[U(y-H)] + g(y)[1 - U(y-H)] \quad \text{at } \theta = 0 \tag{4}$$

$$u = 0 \quad \text{at } y = 0 \tag{5}$$

$$u = U_i \quad \text{as } y \to \infty \tag{6}$$

where θ is the age of the cycle, U_i is the axial velocity of the inrushing fluid, $g(y)$ is the instantaneous velocity distribution within the wall region at the first instant of inrush or renewal, and $U(y-H)$ is a unit step function (see Fig. 1). (For the simplified case in which fluid is assumed to be brought into direct contact with the wall, H is set equal to zero, such that $u = U_i$ at $\theta = 0$.)

Mean transport

The instantaneous transport equations are now transformed into the mean domain. This is the key to the eventual development of laws for u^+ and T^+ that apply to the wall region. By definition, the mean velocity distribution $\bar{u}$ is expressed as

$$\bar{u} = \int_0^\infty P_H(H)\left\{\int_0^\infty P_g(g)\left[\int_0^\infty u\phi(\theta)\mathrm{d}\theta\right]\mathrm{d}g\right\}\mathrm{d}H \tag{7}$$

where the distributions in the random variables θ, s, H and $g(y)$ are represented by $\phi(\theta)$, $P_s(s)$, $P_H(H)$ and $P_g(g)$. The age, frequency, and approach distance

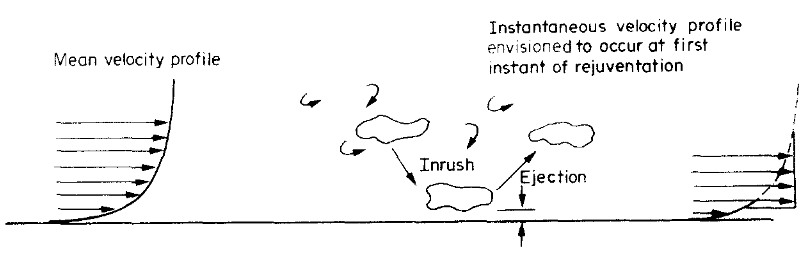

FIG. 1. Picture of the turbulent burst process.

distributions are approximated by (10)

$$\phi(\theta) = \bar{s}\exp(-\bar{s}\theta) \quad (8)$$

$$P_s(s) = -\frac{1}{\bar{s}}\frac{\mathrm{d}\phi(\theta)}{\mathrm{d}\theta} = \bar{s}\exp(-\bar{s}\theta) \quad (9)$$

$$P_H(H) = \frac{1}{\bar{H}}\exp\left(-\frac{H}{\bar{H}}\right). \quad (10)$$

Accordingly, equations (3)–(6) are transformed into the mean domain by multiplying each term by $\phi(\theta)\mathrm{d}\theta$, $P_g(g)\mathrm{d}g$ and $P_H(H)\mathrm{d}H$, and then integrating.

Operating first with respect to θ, equations (3)–(6) are transformed into the form

$$\tilde{u} - g(y)[1 - U(y-H)] - U_i[U(y-H)] = \frac{\nu}{\bar{s}}\frac{\mathrm{d}^2\tilde{u}}{\mathrm{d}y^2} \quad (11)$$

$$\tilde{u} = 0 \quad \text{at} \quad y = 0 \quad (12)$$

$$\tilde{u} = U_i \quad \text{as} \quad y \to \infty \quad (13)$$

where

$$\tilde{u} = \bar{s}\int_0^{\infty} u\exp(-\bar{s}\theta)\mathrm{d}\theta = \bar{s}\mathscr{L}(u) \quad (14)$$

and $\mathscr{L}(u)$ is the Laplace transform with respect to θ of the instantaneous profile u.

Operating next with respect to g, equation (11) becomes

$$\hat{u} - \overline{g(y)}[1 - U(y-H)] - U_i[U(y-H)] = \frac{\nu}{\bar{s}}\frac{\mathrm{d}^2\hat{u}}{\mathrm{d}y^2} \quad (15)$$

where

$$\hat{u} = \int_0^{\infty} P_g(g)\tilde{u}\,\mathrm{d}g. \quad (16)$$

Finally, operating with respect to H, equation (15) and the boundary conditions are transformed into the mean domain.

$$\bar{u} - \overline{g(y)}\exp\left(-\frac{y}{\bar{H}}\right) - U_i\left[1 - \exp\left(-\frac{y}{\bar{H}}\right)\right] = \frac{\nu}{\bar{s}}\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}y^2} \quad (17)$$

$$\bar{u} = 0 \quad \text{at} \quad y = 0 \quad (18)$$

$$\bar{u} = U_i \quad \text{as} \quad y \to \infty \quad (19)$$

The use of equations (8) and (9) result in

$$g(y) = \bar{u} \quad (20)$$

such that equation (17) reduces to

$$(\bar{u} - U_i)\left[1 - \exp\left(-\frac{y}{\bar{H}}\right)\right] = \frac{\nu}{\bar{s}}\frac{\mathrm{d}^2\bar{u}}{\mathrm{d}y^2}. \quad (21)$$

The analytical solution to this type of ordinary differential equation has been obtained by utilizing the substitutions $\psi = \bar{u} - U_i$, $Z = \exp[-y/(2\bar{H})]$, and $\xi = 2Z\bar{H}\sqrt{\bar{s}/\nu}$ [10]. The final solution for the mean velocity distribution takes the form

$$\frac{\bar{u}}{U_i} = 1 - J_{2\gamma}\left[2\gamma\exp\left(-\frac{y}{2\bar{H}}\right)\right]\frac{1}{J_{2\gamma}(2\gamma)} \quad (22)$$

where $\gamma = \bar{H}\sqrt{\bar{s}/\nu}$ and $J_{2\gamma}$ is a Bessel function of the first kind and 2γ order. It follows that the mean wall shear stress $\bar{\tau}_0$ becomes

$$\bar{\tau}_0 = \mu\left.\frac{\mathrm{d}\bar{u}}{\mathrm{d}y}\right|_0 = \mu U_i\sqrt{\frac{\bar{s}}{\nu}}\,\frac{J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)}{2J_{2\gamma}(2\gamma)}. \quad (23)$$

A similar analysis of the energy transfer associated with the burst mechanism gives rise to an expression for the mean temperature distribution within the wall region of the form [10]

$$\frac{\bar{T} - T_0}{T_i - T_0} = 1 - J_{2\beta}\left[2\beta\exp\left(-\frac{y}{2\bar{H}}\right)\right]\frac{1}{J_{2\beta}(2\beta)} \quad (24)$$

where $\beta = \bar{H}\sqrt{\bar{s}/\alpha} = \gamma\sqrt{Pr}$. The mean wall heat flux $\overline{q_0''}$ is given by

$$\overline{q_0''} = -k\left.\frac{\mathrm{d}\bar{T}}{\mathrm{d}y}\right|_0 = k(T_0 - T_i)\sqrt{\frac{\bar{s}}{\alpha}}\,\frac{J_{2\beta-1}(2\beta) - J_{2\beta+1}(2\beta)}{2J_{2\beta}(2\beta)}. \quad (25)$$

Equation (25) has been found to be in excellent agreement with the previous numerical formulation-solutions by Harriott [6], Thomas *et al.* [7], Bullin and Dukler [8] and Rajagopal and Thomas [9].

Specification of modeling parameters

The surface rejuvenation model of wall turbulence involves the four modeling parameters U_i, $\bar{s}$, $\bar{H}$ and T_i. However, to produce inner laws for u^+ and T^+, only two of these parameters need be specified. This point will be expanded upon in the following section.

DEVELOPMENT OF INNER LAWS

The solutions given in the previous section for the mean velocity and temperature distributions within the wall region are now coupled with the classical approach to obtain laws for u^+ and T^+ for the entire inner region.

Inner laws for u^+

To obtain an expression for u^+ near the wall, equations (22) and (23) are combined, with the result

$$u^+ = \frac{2H^+}{\gamma}\,\frac{J_{2\gamma}(2\gamma) - J_{2\gamma}[2\gamma\exp(-y^+/2H^+)]}{J_{2\gamma-1}(2\gamma) - J_{2\gamma+1}(2\gamma)}$$

Hydrodynamic wall region (26a)

Note that by coupling these two equations, U_i has been eliminated, such that only $\bar{s}$ and $\bar{H}$ remain to be specified.

Because of the modeling assumptions employed in the development of equation (26a), this expression only applies to the wall region. To develop a law that extends into the turbulent core, equation (26a) is interfaced with the classical eddy diffusivity approach. The eddy diffusivity is generally approximated by

$$\frac{\varepsilon_m}{\nu} = \kappa y^+ \tag{27}$$

in the intermediate region for fully turbulent flow with small pressure gradients; κ is approximately equal to 0.40 for internal flow and 0.41 for boundary layer flow. With this input for ε_m, the classical approach gives rise to the familiar logarithmic law for u^+ in the intermediate zone.

$$\int \mathrm{d}u^+ = \int \frac{\mathrm{d}y^+}{1 + \varepsilon_m/\nu} \simeq \int \frac{\mathrm{d}y^+}{\kappa y^+}$$

$$u^+ \simeq \frac{1}{\kappa} \ln y^+ + C \quad \text{Turbulent core.} \tag{26b}$$

Whereas κ is strictly dependent upon the eddy transport mechanism within the turbulent core, C is a function of the wall turbulence, and as such, is related to the burst modeling parameters $\bar{s}$ and $\bar{H}$. To close the analysis, two of these three parameters must be specified. Based on the experimental flow visualization study by Popovich and Hummel [11], the dimensionless mean approach distance H^+ is set equal to 5.0. Whereas limited experimental data are available for the mean burst frequency $\bar{s}$, the parameter C is quite well established for fully turbulent conditions ($C \simeq 5.5$ for internal flows; $C \simeq 5.0$ for boundary layer flows). Therefore, C will be specified instead of $\bar{s}$.

To couple the surface rejuvenation model with the classical approach, continuity is required in both u^+ and $\mathrm{d}u^+/\mathrm{d}y^+$ at the point of interface y_M^+ between equations (26a) and (26b). With $C = 5.5$, $\kappa = 0.4$ and $H^+ = 5$, this approximate interfacing criterion is satisfied for a value of γ equal to about 0.433. The point of interface y_M^+ is 35.

Predictions for u^+ given by equation (26) with $\gamma = 0.433$ and $y_M^+ = 35$ are shown together with experimental data in Fig. 2. For purpose of comparison, the surface renewal expression given by equation (1) and the van Driest [4] equation are also shown.

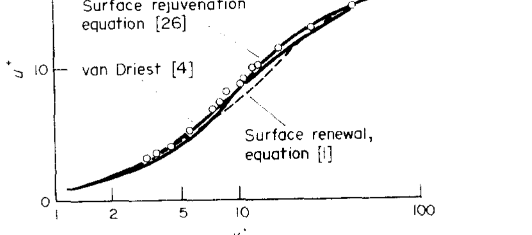

FIG. 2. Inner laws for u^+.

Inner law for T^+

To obtain an expression for the dimensionless temperature distribution T^+ within the wall region, equations (24) and (25) are combined. This step gives

$$T^+ = 2H^+ \frac{Pr}{\beta} \frac{J_{2\beta}(2\beta) - J_{2\beta}\{2\beta \exp[-y^+/(2H^+)]\}}{J_{2\beta-1}(2\beta) - J_{2\beta+1}(2\beta)}$$

$$\text{Thermal wall region} \tag{28a}$$

where $\beta = \gamma\sqrt{Pr}$. Note that no unknowns appear in this equation. Throughout the remainder of the inner region, the classical approach gives

$$\int \mathrm{d}T^+ = \int \frac{\mathrm{d}y^+}{\frac{1}{Pr} + \frac{\varepsilon_m/\nu}{Pr_t}} \tag{29}$$

where the turbulent Prandtl number Pr_t is approximately equal to a constant of the order of unity. Following through with this integration outside the thermal wall region with Pr_t set equal to unity, T^+ is given by

$$T^+ \simeq \frac{1}{\kappa} \ln y^+ + A \quad \text{Turbulent core} \tag{28b}$$

for moderate to high Prandtl number fluids. A is a function of the energy transfer within the wall region, which is fully specified by equation (28a). To approximate A and the point of interface y_T^+, equations (28a) and (28b) are simply coupled at the location at which the gradients $\mathrm{d}T^+/\mathrm{d}y^+$ are equal. With γ set equal to 0.433 on the basis of the analysis of momentum transfer, y_T^+ and A are obtained for various values of Pr. The resulting predictions for y_T^+ and A are given in Fig. 3. The composite inner law for T^+ represented by equation (28) is compared with experimental data and with equation (2) in Fig. 4.

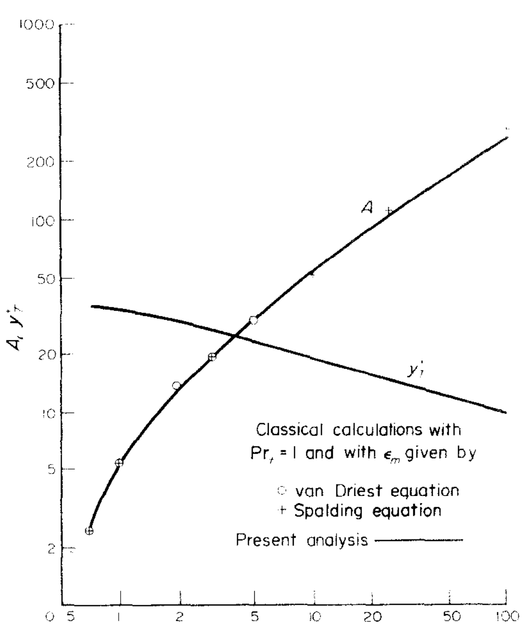

FIG. 3. Predictions for A and y_T^+.

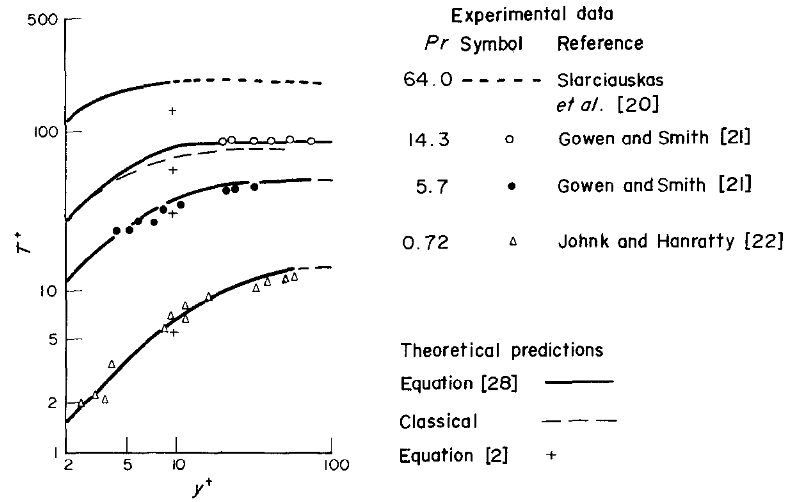

FIG. 4. Inner laws for T^+.

DISCUSSION AND CONCLUSION

Referring to Fig. 2, the present inner law for u^+ given by equation (26) is seen to be in exceptional agreement with the data and with the van Driest equation. The small difference between equations (1) and (26) reflects the effect of the unreplenished layer of fluid on the momentum transfer in the wall region. Similarly, Fig. 4 reveals a good agreement between equation (28) and experimental data for T^+ for Pr as large as 64. The important effect of the unreplenished layer of fluid on heat transfer is reflected in the growing incompatibility between equations (2) and (28) with increasing Pr.

In order to compare equation (28) with classical inner laws for T^+ which are obtained from equation (29), it is necessary to introduce an approximation for the turbulent Prandtl number Pr_t. It is the uncertainty concerning Pr_t or ε_H within the wall region that poses one of the more nagging problems in the classical approach to analysing turbulent convection heat transfer. Because of a general lack of confidence on this issue, the usual practice in the classical approach to turbulence has been to simply set Pr_t equal to a constant of the order of unity (some investigators choose $Pr_t = 0.9$). For the sake of comparison, equation (29) is integrated with ε_m specified by the van Driest equation and with Pr_t set equal to unity. The predictions for A and T^+ obtained by this approach are compared with the results of the present analysis in Figs. 3 and 4. In addition, calculations obtained for A by White [12] on the basis of the Spalding [5] equation for ε_m are shown in Fig. 3.

In view of the uncertainty in Pr_t near the wall, it is particularly interesting to find such a high degree of compatibility between the present analysis and the very simple classical analysis. When confronted with essentially the same problem of explaining the success of the classical analysis with $Pr_t = 1$, Kays [13] concluded that the ε_m he used (after Deissler [14]) may be "truly the eddy diffusivity for heat in the sublayers and that we still do not know accurately the eddy diffusivity for momentum in the sublayers". Spalding [15] put it this way, "... Pr_t is obtained by dividing one uncertain quantity (ε_m) by another (ε_H); and, when it exhibits excessive variability, it is hard to know how to distribute blame ... even when a fixed value of Pr_t appears explicitly in a turbulence model, it will be associated not with the actual turbulence viscosity but with the fictitious one ...; the resulting value of ε_H may be correct, even when the predicted ε_m is not". Given this level of uncertainty in the classical approach, the merit of the surface rejuvenation concept as a supplemental tool for analysing turbulent convection heat transfer stands out. Although this model of the burst phenomenon does not necessitate the use of classical eddy diffusivities or turbulent Prandtl number, this modeling concept recently has been utilized to develop theoretical predictions for ε_m and Pr_t within the wall region [16–18, 23].

As further evidence of the viability of the general surface renewal model of wall turbulence, predictions for the mean frequency of the burst process $\bar{s}$ obtained from equations such as equation (23) have been compared with experimental data for the mean frequency of wall turbulence fluctuations for various applications [23–27]. The predictions for $\bar{s}$ have been found to be fully compatible with the experimental data.

In conclusion, the strength of the model of wall turbulence put forth in this paper is felt to be its sound physical basis and the use of modeling parameters such as $\bar{s}$ and $\bar{H}$ which are measureable. It is believed that this approach will lead to better predictive capabilities for handling turbulent transport in the critical wall region and will help to clear up questions concerning Pr_t.

REFERENCES

1. A. J. Grass, Structural features of turbulent flow over smooth and rough boundaries, *J. Fluid Mech.* **50**, 233 (1971).
2. S. J. Kline, The structure of turbulent boundary layers, *J. Fluid Mech.* **30**, 741 (1967).
3. L. C. Thomas, The turbulent burst phenomenon: inner laws for u^+ and T^+, NATO Advanced Study Institute in Turbulence, Istanbul (1978).
4. E. R. van Driest, On turbulent flow near a wall, *J. Aeronaut. Sci.* **23**, 1007 (1956).
5. D. B. Spalding, A single formula for the law of the wall, *J. Appl. Mech.* **28**, 455 (1961).
6. P. Harriott, A random eddy modification of the penetration theory, *Chem. Engng Sci.* **17**, 149 (1962).
7. L. C. Thomas, B. T. F. Chung and S. K. Mahaldar, Temperature profiles for turbulent flow of high Prandtl number fluids, *Int. J. Heat Mass Transfer* **14**, 1465 (1971).
8. J. A. Bullin and A. E. Dukler, Random eddy modification for surface renewal: formulation as a stochastic process, *Chem. Engng Sci.* **27**, 439 (1972).
9. R. Rajagopal and L. C. Thomas, Adaptation of the stochastic formulation of the surface rejuvenation model to turbulent convective heat transfer, *Chem. Engng Sci.* **29**, 1639 (1974).
10. L. C. Thomas, P. J. Gingo and B. T. F. Chung, The surface rejuvenation model for turbulent convective transport—an exact solution, *Chem. Engng Sci.* **30**, 1239 (1975).
11. A. T. Popovich and R. L. Hummel, Experimental study of the viscous sublayer in turbulent pipe flow, *A.I.Ch.E. J.* **13**, 854 (1967).
12. F. M. White, *Viscous Fluid Flow*, McGraw Hill, New York (1974).
13. W. M. Kays, *Convective Heat and Mass Transfer*, McGraw Hill. New York (1966).
14. R. G. Deissler, NACA Rept. 1210, Washington D.C. (1955).
15. D. B. Spalding, Turbulence models for heat transfer, *Proc. of 6th Int. Heat Transfer Conference*, Vol. 6, p. 33, Toronto (1978).
16. R. Rajagopal and L. C. Thomas, The formulation of relationships for ε_m and ε_H based on a physically realistic model of turbulence, *Proc. 5th Int. Heat Transfer Conf.* Tokyo (1974).
17. L. C. Thomas, A formulation for ε_m and ε_H based on the surface renewal principle, *A.I.Ch.E. J.* **24**, 101 (1978).
18. L. C. Thomas and M. B. Ibrahim, A model of the turbulent burst phenomenon: Predictions for the turbulent Prandtl number, ASME Winter Annual Meeting, New York (1979).
19. E. R. Lindgren, Oklahoma State University, Civil Engng Dept. Rept., IAD 621071 (1965).
20. A. A. Slarciauskas, A. A. Pedisius, A. A. Zukauskas, Universal profiles of the temperature and turbulent Prandtl number in the boundary layer on a plate in a stream of fluid, *Trudy Akad. Nauk Lit. SSR*, Ser. B, 2 (1971) Eng. transl. *Heat Transfer Soviet Res.* **5**, 623 (1973).
21. R. A. Gowen and J. W. Smith, The effect of the Prandtl number on temperature profiles for heat transfer in turbulent pipe flow, *Chem. Engng Sci.* **22**, 1701 (1967).
22. R. E. Johnk and T. J. Hanratty, Temperature profiles for turbulent flow of air in a pipe—I. The fully developed heat transfer region, *Chem. Engng. Sci.* **17**, 867 (1962).
23. L. C. Thomas and M. B. Ibrahim, A surface rejuvenation model of the turbulent burst phenomenon: Predictions for ε_m, UPM Report (1980).
24. P. M. Gerhart and L. C. Thomas, A model of turbulent momentum and heat transfer at points of separation or reattachment, *Proc. Heat Transfer Fluid Mech. Inst.*, p. 122. Stanford University Press, Palo Alto (1974).
25. L. C. Thomas and H. L. Greene, An experimental and theoretical study of the viscous sublayer for turbulent tube flow, Symposium on Turbulence in Liquids, The University of Missouri, Rolla (1973).
26. L. C. Thomas and J. Min, An experimental and theoretical study of transitional turbulent pulsatile flow in a channel, *J. Appl. Mech.* **43**, 559 (1976).
27. R. L. Meek and A. D. Baer, The periodic viscous sublayer in turbulent flow, *A.I.Ch.E. Jl* **16**, 841 (1970).

LE MODELE DE LA TURBULENCE DE PAROI PAR RENOUVELLEMENT A LA SURFACE: LOIS INTERIEURES POUR u^+ et T^+

Résumé—Une formulation du modèle de renouvellement en surface par des bouffées turbulentes est couplée à l'approche classique du noyau turbulent pour obtenir les lois intérieures pour la vitesse u^+ et la température T^+ toutes deux adimensionnelles, dans le cas des écoulements pleinement turbulents. La prévision de u^+ est en bon accord avec la loi connue de van Driest. De plus, celle de T^+ s' accorde rrès bien avec les données expérimentales dans une large gamme de nombres de Prandtl.

DAS MODELL DER WANDTURBULENZ MIT ERNEUERUNG DER OBERFLÄCHE: INNERE GESETZMÄßIGKEITEN FÜR u^+ UND T^+

Zusammenfassung—Eine wichtige Beschreibung des Oberflächenerneuerungsmodells für den Vorgang beim Zerplatzen eines Wirbels wird mit der klassischen Näherung für den turbulenten Kern gekoppelt, um innere Gesetzmäßigkeoten für die Verteilung der dimensionslosen Geschwindigkeit u^+ und Temperatur T^+ für ausgeprägte turbulente Strömungen herzustellen. Die Vorhersagen für u^+ werden in einer guten Übereinstimmung mit den wohlbekannten inneren Gesetzmäßigkeiten von van Driest gefunden. Darüberhinaus wurde auch gezeigt, daß sich die Vorhersagen für T^+ in einer vorzüglichen Übereinstimmung mit Versuchswerten für einen großen Bereich von Prandtl-Zahlen befinden.

МОДЕЛЬ ОБНОВЛЕНИЯ ПОВЕРХНОСТИ В СЛУЧАЕ ПРИСТЕННОЙ ТУРБУЛЕНТНОСТИ: ВНУТРЕННИЕ ЗАКОНОМЕРНОСТИ РАСПРЕДЕЛЕНИЯ u^+ И T^+

Аннотация — Одна из эффективных формулировок модели обновления поверхности для описания процесса турбулентного выброса объединена с классическим описанием турбулентного ядра с целью выявления внутренних закономерностей распределений безразмерных скорости u^+ и температуры T^+ при полностью развитом турбулентном течении. Найдено, что расчетные значения u^+ хорошо описываются известным законом ван Дриста. Кроме того показано, что расчетные значения T^+ хорошо согласуются с экспериментальными данными в широком диапазоне значений числа Прандтля.